A Quasi-Coherent Sampling Method for Wideband Data Acquisition

Masaru KIMURA†, Kensuke KOBAYASHI††, Nonmembers, and Haruo KOBAYASHI†††, Regular Member

SUMMARY This paper proposes a quasi-coherent equivalent-time sampling method to acquire repetitive wideband waveform signals with high throughput. We have already proposed a new sampling system which incorporates the pre-trigger ability and the time jitter reduction function for a fluctuated input signal while maintaining the waveform recording efficiency. The quasi-coherent sampling method proposed in this paper can be adopted to it in order to improve its data acquisition throughput significantly. Numerical simulation results show effectiveness of our proposed method.

**key words:** equivalent-time sampling, coherent sampling, Brocot space, high time-resolution, data recording time

1. Introduction

Equivalent-time sampling is a well-known technique to capture repetitive signals at finer time intervals than a sampling clock period and it is widely used in waveform measurement systems with high time-resolution (such as a wideband digital storage oscilloscope (DSO)). There are three techniques for implementing its time base (i.e., sequential sampling, random sampling [1] and coherent sampling [2]), and they have their respective advantages and disadvantages. In [3], we have already proposed a new sampling system which incorporates the pre-trigger ability and the time jitter reduction function for a fluctuating input signal which a random sampling system has, while maintaining the waveform recording efficiency which a conventional coherent sampling system has. Hence, the sampling system proposed in [3] has characteristics of both random and coherent samplings, which would be useful to implement wide-band high-throughput DSOs. However improvements in the coherent sampling part still need to be made. In this paper we propose a new concept of “quasi-coherent sampling” which will further improve data sampling throughput taking into account the time required for changing a clock period.

In Sect. 2, our previously proposed method of coherent sampling with a time interpolator [3] is briefly reviewed. In Sect. 3, its problem is addressed and our new concept of “quasi-coherent sampling” is introduced to overcome it. In Sect. 4, we redefine the three sampling states (coherent, quasi-coherent and incoherent sampling states) to enable to determine the sampling state quantitatively. In Sect. 5, a new algorithm using the Brocot space theory is proposed to determine effectively whether the sampling is in the quasi-coherent state or not, and in Sect. 6, the validity of the algorithm is shown by numerical simulation. In Sect. 7, the numerical simulation results show the effectiveness of our sampling method as a whole: its throughput is approximately twice compared to the previously proposed method. Section 8 provides conclusion.

2. Coherent Sampling with a Time Interpolator

Before discussing the main subject, we will describe the operation of a coherent sampling method with a time interpolator [3] in Fig. 1 and its relational expression in Eq. (1). Also we will define some terms to be used. Figure 1 shows that a repetitive input signal waveform is measured with a time resolution of one-third of the clock period, and three time bins are provided in a clock period. Here, data is sampled at 8 points per two signal periods. $T_c$ is a clock period and $Thold$ is duration of a holdoff, which becomes valid (high) incident to the trigger input and inhibits the next trigger input until it becomes invalid (low). $Tsi$ ($i=1,2,3,..$) is a time difference between the time at which the holdoff is generated and a subsequent clock-edge-rising-timing, and it is called “time interpolation data.” A holdoff period $Trr$ is expressed by Eq. (1) and is obtained by measuring the values of the successive $Tsi$’s and counting the number of clock periods.

$$Trr=Tc\times\text{Ceiling}(Thold/Tc)=K\times T_c – δTsi$$  (1)

where, Ceiling[$x$] is a minimum integer equal to or greater than $x$, and $K$ is the number of clock periods within $Trr$. Also $δTsi$ (defined as $δTsi=Tsi+i–Tsi$) is the time difference between successive $ith$ and $(i+1)$th time interpolation data.

Note that the state where the condition of Eq. (2) holds between $Tc$ and $Trr$ is called a “coherent sampling state.”

$$Tc/Trr=M/N$$  (2)

where, $N$ (=8 in Fig. 1) is the number of clock periods required to collect data, $M$ (=3 in Fig. 1) is the number of time bins, which corresponds to the amount of data sampled per one clock period, and $N$ and $M$ are relatively prime integers.
3. Quasi-Coherent Sampling

When we adopt the above-mentioned coherent sampling method for a repetitive input signal whose period fluctuates, the period of the sampling clock has to be adjusted according to the input signal period. However, the settling time of a very stable oscillator (e.g. its \( \text{rms} \) jitter < 0.5ps implemented with a voltage controlled crystal oscillator (VCXO), which is required for high time-resolution measurement) is relatively long (several tens of milliseconds in this example) and hence the system throughput for data acquisition is degraded. Also, precise oscillation frequency adjustment to satisfy the coherent condition is very difficult. This is because the “control voltage versus oscillation frequency” of the VCXO has non-linear characteristics and an oscillator using PLL has a finite frequency setting resolution. To overcome this problem, we will consider and define three sampling states for the relationships between the sampling clock period \( T_c \) and the holdoff period \( T_{rr} \) which is a multiple of the input signal period:

1. **Coherent sampling state:** \( T_{rr} \) and \( T_c \) satisfy the relationships in Eq. (2). In this case, the efficiency of obtaining time interpolation data of all time bins is very high (=100%).

   **Definition of “Coherent Sampling”** If all of \( M \) time bins are filled out with \( M \) samples, it is defined to be in the coherent sampling state.

2. **Quasi-coherent sampling state** (newly introduced concept): The relationships between \( T_{rr} \) and \( T_c \) are moderately appropriate and “bunched waveform phenomena” [4], [7], [8] do not occur. In such a state, the efficiency may not be as high as in the coherent state, but it is fairly high and the period of the sampling clock needs not to be changed.

   **Definition of “Quasi-Coherent Sampling”** If \( M \) time bins are filled out with \( M \) samples by \( n \times M \) times where \( n \) is greater than one and is a reasonably small positive number (\( n=1.375 \) in this example), it is defined to be in the quasi-coherent sampling state.

3. **Incoherent sampling state:** The relationships between \( T_{rr} \) and \( T_c \) are not appropriate and bunched waveform phenomena occur. Hence in this case the efficiency is not good and the period of the sampling clock should be changed.

   **Definition of “Incoherent Sampling”** If \( M \) time bins are not filled out with \( M \) measurements by \( n \times M \) times where \( n>1 \) and \( n \) is a reasonably small positive number, it is defined to be in the incoherent sampling state.

**Remark:** Figure 3 shows an example of the bunched waveform phenomena or waveform missing phenomena in a DSO for a sinusoidal input signal. We see that some parts of the sinusoidal signal is not displayed in a DSO, and this happens when the input signal repetition period and the holdoff period \( T_{rr} \) satisfy some conditions. The details of their mechanism and countermeasures are described in [4], [7], [8].
For the coherent sampling at the top in Fig. 2, since $\delta T_{\text{si}}/T_c = \text{(odd integer)}/8$, (that is, the numerator and denominator are relatively prime integers), it is possible to determine quantitatively using $\delta T_{\text{si}}/T_c$, whether it is in the coherent sampling state. However, for the quasi-coherent sampling state, it is not obvious. Thus, we will redefine these sampling states as follows to enable to determine quantitatively whether it is in the quasi-coherent sampling state:

**Redefinition of “Coherent Sampling”** Collect time interpolation data $T_{\text{si}}$ by $M=8$ in Fig. 2 times and put the data in the ascending order. If a maximum value $\Delta T_{s, \text{max}}$ of the difference between adjacent terms in this progression is not more than $T_c/M$, then it is defined as in the coherent sampling state.

**Redefinition of “Quasi-Coherent Sampling”** Collect $T_{\text{si}}$ by $n\times M$ ($n \geq 1$) times and put their data in the ascending order. If a maximum value $\Delta T_{s, \text{max}}$ of the difference between adjacent terms in this progression is not more than $T_c/M$, it is defined as in the quasi-coherent sampling state.

Figure 4 shows $\delta T_{\text{si}}/T_c$ (on the $x$-axis (1/4 $\leq x \leq 1/2$) versus $\Delta T_{s, \text{max}}/T_c$ (on the $y$-axis) for $M=8$ and $n=1, 2, 3$. With an $x$ value in the area of $y > 1/M$ ($=1/8$), there are some time bins in which data are not filled during $n\times M$ time interpolation data samplings, which yields to be in the incoherent sampling state.

1. In case $n=1$ (8 times of $T_{\text{si}}$ samplings): For $y=1/8$ at $x=3/8$, it is in the coherent sampling state. With other $x$ values, $y > 1/8$ and there are some time bins where data are not filled out.
2. In case $n=2$ or 3 (16 or 24 times of $T_{\text{si}}$ samplings): There are some quasi-coherent sampling areas for $y < 1/8$, and there all time bins are filled out by data with $n\times M$ times $T_{\text{si}}$ measurements. Note that the peaks in the area of $y \geq 1/8$ coincide with $n=1$ case.

**Fig. 3** Example of bunched waveform phenomena or waveform missing phenomena for a sinusoidal input in a DSO. Some waveform parts are missing in the DSO display when the input signal period and the holdoff period satisfy some conditions.

**Fig. 4** Redefinition of the coherent sampling state (regions of $\delta T_{\text{si}}/T_c$’s where $\Delta T_{s, \text{max}}/T_c=1/M$ (i.e., $y=1/8$)), the quasi-coherent sampling state (regions of $\delta T_{\text{si}}/T_c$’s where $\Delta T_{s, \text{max}}/T_c < 1/M$), and the incoherent sampling state (regions of $\delta T_{\text{si}}/T_c$’s $> 1/M$ where $\Delta T_{s, \text{max}}/T_c > 1/M$).

**Fig. 5** Coherent ($n=1$) and quasi-coherent ($n>1$) sampling regions with respect to $\delta T_{\text{si}}/T_c$ value. Dots and line segments show the regions of $\delta T_{\text{si}}/T_c$’s values for $n=1, 1.5, 2, 2.5, 3$ and $M=8$, where the sampling is in a coherent or quasi-coherent state.

Dots and line segments in Fig. 5 show the regions of $\delta T_{\text{si}}/T_c$’s values for $n=1, 1.5, 2, 2.5, 3$ and $M=8$, where $\Delta T_{s, \text{max}}/T_c$ is equal to or less than $1/M$ (i.e. the sampling is in the coherent or quasi-coherent state). We have calculated $\delta T_{\text{si}}/T_c$’s from 0 to 1 by a 1/256 step, and the difference between $n=1$ (coherent) and $n>1$ (quasi-coherent) lies in that in the former case, $\delta T_{\text{si}}/T_c$’s are located as “points” such as $\delta T_{\text{si}}/T_c = 1/8, 3/8, 5/8 and 7/8$, while in the latter case they are as “line segments”; this is a nice feature of the quasi-coherent sampling, presenting the ability to maintain a constant clock period regardless of setting errors of the clock period and slight fluctuations in the signal period, and this is why we have introduced here the new concept of “quasi-coherent sampling state.” Figures 6 shows an example of captured waveforms in the coherent, quasi-coherent and incoherent sampling states to help the reader to understand the three sampling states intuitively.

However, in some practical applications, it takes considerable time to calculate $\Delta T_{s, \text{max}}$. For example, with a wideband data sampling apparatus with a high time-resolu-
tion of 1ps at a 100 MHz clock rate, \(M\) amounts to 10,000 (=10 ns/1 ps). To determine whether the quasi-coherent state condition for \(n=2\) is satisfied or not, it is required to generate 20,000 \(Tsi\)'s, put them in the ascending order, calculate differences between adjacent terms and find their maximum value. To overcome this calculation complexity, we will propose a new algorithm which simplifies this calculation significantly in the next section.

5. Brocot Space and Expansion of Continued Fractions

A Brocot space [5] is known as a means of illustrating whether the operation of a sampling system is extremely close to a coherent sampling state or not [6]. We consider that this space is also effective in determining whether the sampling is in the quasi-coherent state, and we will show an example with \(M=8\) and \(n=2\):

Figure 7 shows irreducible fractions in \(n \times M\) th (=16th) order Brocot space in 1/4–1/2 range sequentially connected with solid lines. We see that these solid lines have the same shape as that of the lines for \(n=2\) in Fig. 4. Similarly, the lines for \(n=1\) in Fig. 4 match the dotted lines connecting irreducible fractions in up-to-8th order Brocot space in Fig. 7. In general, \(\Delta T_{\text{max}}/Tc\) can be obtained by sequentially connecting fractions in the \(n \times M\) th order Brocot space when \(n \times M\) time interpolation data pieces are sampled for each \(\delta Tsi/Tc\). From these observations, we have obtained the followings:

1. If the denominator of \(\delta Tsi/Tc\) takes a value between \(M\) and \(n \times M\), it is in the coherent or quasi-coherent sampling state.
2. If the denominator of \(\delta Tsi/Tc\) takes a value less than \(M\), it results in the incoherent sampling state.
3. If \(\Delta T_{\text{max}}/Tc\) of \(\Delta Tsi/Tc\) has a denominator larger than \(n \times M\), it takes a value between the \(\Delta T_{\text{max}}/Tc\)'s of two adjacent irreducible fractions not greater than \(n \times M\).

Any rational number can be expressed by an irreducible fraction. Therefore, a simple evaluation method of \(\delta Tsi/Tc\) whether its denominator is greater than \(n \times M\) would enable to quickly determine, for arbitrary \(\delta Tsi\), whether the sampling is in the quasi-coherent state. As a trial, we have expanded \(\delta Tsi/Tc\) into continued fractions expressed by Eq. (3) and focused on denominator(s) \(q_n\)'s of convergents expressed by Eq. (4).

\[
\frac{\delta Tsi}{Tc} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \cdots + \frac{1}{a_n}}}
\]  
(3)

\[
q_0 = a_0 q_0 = a_1 q_0 + a_2 q_1 = a_2 q_2 + a_3 q_0 + a_4 q_1
\]  
(4)

As a result, we have observed one regular pattern. Figure 8 shows \(\Delta T_{\text{max}}/Tc\) versus \(\Delta Tsi/Tc\) with a denominator 43 of \(\delta Tsi/Tc\) expressed by black circles on the line segments of \(n=2\) in Fig. 4. The first and third \(\delta Tsi/Tc\)'s from the left of the section enclosed by dotted lines do not satisfy a quasi-coherent sampling state, while the second, fourth, fifth and sixth \(\delta Tsi/Tc\)'s satisfy it.

Figure 9 shows plots of convergents values obtained by expanding the above 6 \(\delta Tsi/Tc\)'s into continued fractions. The rectangular blocks of dotted lines indicate positions of frac-
tions whose denominators are 8 and 16. $\delta T_{si}/T_c$'s with a denominator of 43 circled by dotted lines (15/43, 17/43) reach their final values without passing the convergents inside the rectangular block in expansion of continued fractions. On the other hand, $\delta T_{si}/T_c$'s with a denominator of 43 circled by solid lines (16/43, 18/43, 19/43, 20/43) reach their final values through the convergents inside the rectangular block in expansion of continued fractions. From the above observations, we have deduced the following hypothesis:

“If a convergent denominator of $\delta T_{si}/T_c$ passes between $M$ and $n \times M$, the sampling corresponding to such a value of $\delta T_{si}/T_c$ is in the quasi-coherent state.”

6. Verification of Hypothesis

Supposing $n=2$, we have performed numerical calculations of $\Delta T_{s_{\text{max}}}/T_c$ for various types of $M$ and $\delta T_{si}/T_c$ to determine whether the quasi-coherent sampling state is in place. And also we have conducted numerical calculations to see whether the denominator of a convergent of each $\delta T_{si}/T_c$ would pass between $M$ and $n \times M$. The followings are the results of calculations for 4,999 samples of $\delta T_{si}/T_c$’s whose values take from 0 to 1 and for $M=128$:

“Area A” indicates the occurrence rate (with respect to $M$) that the convergent denominator of $\delta T_{si}/T_c$ passes between 128 and 256 in the quasi-coherent (or coherent) sampling state. For our simulation with $n=2$ and $M=128$, the number of such $\delta T_{si}/T_c$’s is 2,272 (i.e. its occurrence rate is 2,272/4,999).

“Area B” shows the occurrence rate that the convergent denominator of $\delta T_{si}/T_c$ does not pass between 128 and 256 in the quasi-coherent (or coherent) sampling state. For our simulation with $n=2$ and $M=128$, the number of such $\delta T_{si}/T_c$’s is 396 (i.e. its occurrence rate is 396/4,999).

“Area C” shows the occurrence rate that the convergent denominator of $\delta T_{si}/T_c$ passes between 128 and 256 in the incoherent sampling state. For our simulation with $n=2$ and $M=128$, the number of such $\delta T_{si}/T_c$’s is 0 (i.e. its occurrence rate is 0).

“Area D” shows the occurrence rate that the convergent denominator of $\delta T_{si}/T_c$ does not pass between 128 and 256 in the incoherent sampling state. For our simulation with $n=2$ and $M=128$, the number of such $\delta T_{si}/T_c$’s is 2,331 (i.e. its occurrence rate is 2,331/4,999).

Fig. 8 $\Delta T_{s_{\text{max}}}/T_c$ versus $\delta T_{si}/T_c$ when a denominator of $\delta T_{si}/T_c$ is 43.

Fig. 9 Convergents plots of fractions for a denominator 43 of $\delta T_{si}/T_c$ shown in Fig. 8.

Fig. 10 Simulation results for occurrence rates of sampling states for $\delta T_{si}/T_c$; Area A and Area D are judged as the quasi-coherent and incoherent sampling states respectively, both by convergent and $\Delta T_{s_{\text{max}}}/T_c$ methods. However Area B is judged as the quasi-coherent sampling state by the $\Delta T_{s_{\text{max}}}/T_c$ method but judged as the incoherent sampling state by the convergent method. Note that the occurrence rate for Area C is zero and hence it does not appear in the graph.
Figure 10 shows results of the occurrence rates (Y-axis) of each area with respect to \( M \) (note that X-axis is the logarithmic scale of \( M \)). The results show that the occurrence rate is zero in area C and it is a little bit less than 10% in area B. Thus we see that \( \delta Tsi/Tc \) which reaches a final value with the convergent denominator passing between \( M \) and \( n \times M \) satisfies a sufficient condition to be “in the quasi-coherent sampling state.”

7. Simulation of Waveform Recording Times

We have conducted numerical calculation of the waveform recording time for the random sampling, the coherent sampling and the quasi-coherent sampling systems in order to evaluate the effectiveness of our proposed method. The calculation conditions are as follows:

1. The total number of data pieces to be recorded is set to 1,024. The number of interpolation points \( M \) per one clock period is set to 512 (supposing the clock rate to be 100 MHz, this condition corresponds to 2 ns/div. of sweep-time and 20 ps of time-resolution). Waveform data is recorded after the trigger input time point, and we calculate the waveform recording time by repeating the sampling operation until the total data is recorded twice and averaging the required time. If the total data are not recorded during operation of \( 200 \times M \) time-interpolations, sampling operation is suspended and the value obtained by dividing the total recording time by the number of the recorded data pieces is used.

2. The period required for time-interpolation is set to \( 8 \times Tc \), and data transfer time is set to \( 8 \times Tc/\text{data} \). We also assume that the required time to randomize the clock phase in the random sampling system and to change the clock period in the coherent and the proposed sampling system are \( 100,000 \times Tc \) (1 ms if the clock rate is 100 MHz).

3. We suppose that the clock period (or reference period) is 1, and the clock is ideal (without fluctuations in time periods). Also we assume that the input signal has fluctuations with a Gaussian distribution in time periods. The average time period of the target input signal has 21 levels (in 0.45 steps) in a range of 1 to 10 and the standard deviation of its fluctuations has 5 levels in a range of \( 10^{-2} \) to \( 10^{-6} \).

4. The number of accumulation times to find the hold-off period \( Trr \) in the coherent and the proposed sampling systems is 128, and both systems adopt a constant hold-off method [7], [8] to avoid waveform missing phenomena.

Figure 11 shows numerical calculation results, which lead to the following observations:

1. In the coherent and the proposed quasi-coherent sampling systems, the waveform recording time (z-axis value) is short when signal fluctuation (y-axis value) is small, while it is long when signal fluctuation is large. Its reason is as follows: when the input signal fluctuation is small, the system performs a coherent or a quasi-coherent sampling operation and records waveform data with high efficiency. On the other hand, when the fluctuation is large, the system ends up recording waveform data randomly no matter how the clock period is controlled.

2. In the random sampling, the waveform recording time is almost independent of both the average period and the standard deviation of the input signal period fluctuation. However, the waveform recording time is quite long due to the clock period randomizing scheme.

Figure 12 shows the simulation results comparison among the three sampling methods. X-axis indicates the standard deviation of the input signal period fluctuation while y-axis shows the mean value of the waveform recording time of 21 levels for the input signal period. We see that the proposed sampling system can record data with time less than \( 1/2 \) of that of the coherent sampling system. Its main reason is that \( \delta Tsi \)'s which satisfy the quasi-coherent sampling state of \( n=2 \) amount to 50% of all \( \delta Tsi \)'s, as is observed in Fig. 10. Therefore the number of times of the clock period changes reduces almost by half.

Also note that the proposed and the coherent sampling systems can record data with time less than \( 1/100 \) of that of
8. Conclusion

We have proposed a new quasi-coherent sampling method for wideband waveform measurement which improves high waveform recording efficiency significantly. First a new concept of “quasi-coherent sampling” is introduced, and next it is formulated to determine quantitatively whether the sampling is in the quasi-coherent state. Then we propose an effective algorithm using the Brocot space theory to determine it. The numerical simulation results showed that the proposed sampling system can record data with more than twice faster compared to a coherent sampling system with a time interpolator which we previously proposed. Hence the proposed method would be effective for implementing high performance DSOs.

References


Masaru Kimura received the B.S. degree in Electronic Engineering from Saitama University in 1986. In 1986 he joined IWATSU Electric Co. Ltd., where he was engaged in research of superconductive sampler and design of ASICs for oscilloscopes. From 1997 to 2001 he was involved in the research and development of high-speed analog-to-digital converters and data acquisition systems in Teratec Corporation. Since 2001, he has been designing of IC for analyzing system in IWATSU.

Kensuke Kobayashi received the B.S. degree in electronic engineering from the University of Tokyo in 1971. In 1971 he joined IWATSU Electric Co. Ltd., where he was engaged in designing wideband sampling oscilloscopes. From 1993 to 2001, he joined Teratec Corporation and developed fast-sampling-rate and wideband data acquisition systems. In 2001 he joined LeCroy Corp. and is involved in ultra wideband oscilloscope research.

Haruo Kobayashi received the B.S. and M.S. degrees in information physics from University of Tokyo in 1980 and 1982 respectively, the M.S. degree in electrical engineering from University of California at Los Angeles (UCLA) in 1989, and the Dr. Eng. degree in electrical engineering from Waseda University in 1995. He joined Yokogawa Electric Corp. Tokyo, Japan in 1982, where he was engaged in the research and development related to measuring instruments and minicomputers. From 1994 to 1997 he was involved in the research and development of ultra-high-speed ADCs and DACs at Teratec Corporation. In 1997 he joined Gunma University and presently is an Associate Professor in Electronic Engineering Department there. He was also an adjunct lecturer at Waseda University from 1994 to 1997. His research interests include analog & digital integrated circuit design and signal processing algorithms. He is a recipient of the 1994 Best Paper Award from the Japanese Neural Network Society.