Analysis of Coupled Inductors for Low-Ripple Fast-Response Buck Converter

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Research Purpose


Coupled inductors in multiphase DC-DC converter.

Clarification of its characteristics in theory, simulation and experiments.

Tradeoff between Efficiency and Transient-Response

- Small Inductor
  - Large current ripple
  - Fast response

- Large Inductor
  - Small current ripple
  - Slow response

- Coupled Inductor
  - Small current ripple
  - Fast response

DC-DC Buck Converter

\[ \Delta I_{L_{Top}} = \frac{V_{in} - V_{out}}{L} T_{on} \]

\[ \Delta I_{L_{Bottom}} = \frac{-V_{out}}{L} T_{off} \]

\[ V_{out} = \frac{T_{on}}{T + T_{off}} V_{in} \]

\[ V_{out} = \frac{T_{on}}{T} V_{in} \]

Coupled Inductor in Two-phase DC-DC Buck Converter

\[ V_i = L \frac{di}{dt} + M \frac{di_2}{dt} \]

\[ V_2 = L \frac{di_2}{dt} + M \frac{di_1}{dt} \]

Mutual inductance:

\[ M = k \times L \quad (-1 \leq k \leq 0) \]

Fig. 1. Two-phase coupled-inductor buck converter

Fig. 2. Operating waveforms of interleaved two-phase coupled inductor buck converter (Duty-cycle \( D \leq 0.5 \))
Steady-state Analysis

Derivation of Steady-state Average Equivalent Inductance

Mode 1:
\[ V_i = \frac{L^2 - M^2}{L + M \cdot \frac{D}{1-D}} \frac{d_i}{dt} \quad \Rightarrow \quad L_{eq1} = L \left( 1 - k^2 \right) \left( 1 + k \cdot \frac{D}{1-D} \right) \]

Mode 2:
\[ V_i = (L + M) \frac{d_i}{dt} \quad \Rightarrow \quad L_{eq2} = L(1 + k) \]

Mode 3:
\[ V_i = \frac{L^2 - M^2}{L + M \cdot \frac{D}{1-D}} \frac{d_i}{dt} \quad \Rightarrow \quad L_{eq3} = L \left( 1 - k^2 \right) \left( 1 + k \cdot \frac{D}{1-D} \right) \]

Average inductor current in one switching period:
\[ \Delta i_t = \frac{\bar{V}_i \cdot D}{L_{eq1}} + \frac{\bar{V}_i \cdot (1-2D)}{L_{eq2}} + \frac{\bar{V}_i \cdot 2D}{L_{eq3}} \]

\[ \bar{V}_i = \frac{\bar{V}_i}{L_{eq1}} + \frac{\bar{V}_i \cdot (1-2D)}{L_{eq2}} + \frac{\bar{V}_i \cdot 2D}{L_{eq3}} \]

- Duty-cycle \( D \leq 0.5 \) \( L_{eq1} \approx L \) \( \Rightarrow \) Low ripple
- Duty-cycle \( D \geq 0.5 \) \( L_{eq3} \approx L \) \( \Rightarrow \) Low ripple

Current Ripple

- Peak to peak current ripple (Duty-cycle \( D \leq 0.5 \)):
\[ I_{pp-coupled} = \frac{d_i}{dt} \times T_{off} \]
\[ = (1 - D)T \times \frac{V_{out}}{L_{eq1}} \]

Minimum current ripple Vs Coupling coefficient
- Derivative functions with \( A = D/(1-D) \):
\[ F = \frac{L_{eq}}{L} = \frac{1 - k^2}{1 + Ak} \]

<table>
<thead>
<tr>
<th>Duty cycle</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
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<tbody>
<tr>
<td>( k )</td>
<td>-0.056</td>
<td>-0.128</td>
<td>-0.225</td>
<td>-0.382</td>
<td>-1.00</td>
</tr>
</tbody>
</table>

Current Ripple Reduction

- Per-phase current ripple reduction:
\[ \frac{|I_{pp-uncoupled} - I_{pp-coupled}|}{I_{pp-coupled}} = -k \left[ k + \frac{D}{1-D} \right] \]

Coupling Coefficient Vs Ripple Reduction for \( D = 40\% \)

<table>
<thead>
<tr>
<th>Ripple Reduction [%]</th>
<th>-35</th>
<th>-30</th>
<th>-25</th>
<th>-20</th>
<th>-15</th>
<th>-10</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
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</thead>
<tbody>
<tr>
<td>Coupling Coefficient ( k )</td>
<td>-1</td>
<td>-0.8</td>
<td>-0.6</td>
<td>-0.4</td>
<td>-0.2</td>
<td>0</td>
<td></td>
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</tbody>
</table>

Transient Analysis

- Output voltage in real circuits:
\[ V_{out} = (V_{in} - I_o \cdot R_s)D. \]

- Current change \( \Delta i \) in coupled inductor circuits:
\[ \Delta i = \frac{V_{in} \cdot T}{L_{eq2}} \Delta D. \]

- \( L_{eq2} = L(1 + k) \ll L \)

Fast transient

Fig. 3. Comparison of load current transient response of buck converters with and without inductor coupling.
Summary

- Analysis of coupled inductors in multiphase DC-DC buck converter.
  - Lower per-phase ripple current
  - Reduced switching losses
  - Faster transient response

Clarification by theory analysis, simulation and experiments.