Explicit Transfer Function of RC Polyphase Filter for Wireless Transceiver Analog Front-End

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Research Goal

- To establish systematic design and analysis methods of RC polyphase filters.

- As its first step, to derive explicit transfer functions of the 1st-, 2nd- and 3rd-order RC polyphase filters.
Roles of RC Polyphase Filter in Wireless Transceiver
Features of RC Polyphase Filter

- Its input and output are complex signal.
- Passive RC analog filter
- One of key components in wireless transceiver analog front-end
  - I, Q signal generation
  - Image rejection
- Its explicit transfer function has not been derived yet.
First-order RC Polyphase Filter

\[ V_{in} = I_{in} + jQ_{in} \]

\[ V_{out} = I_{out} + jQ_{out} \]

Differential Complex Input: \( V_{in} = I_{in} + jQ_{in} \)

Differential Complex Output: \( V_{out} = I_{out} + jQ_{out} \)
I, Q signal generation from single sinusoidal input

\[ I_{\text{in}} = \cos \left( \omega_{\text{LO}} t \right) \]

Polyphase Filter

\[ I_{\text{out}} = A \cos \left( \omega_{\text{LO}} t + \theta \right) \]

\[ Q_{\text{in}} = 0 \rightarrow Q_{\text{out}} = A \sin \left( \omega_{\text{LO}} t + \theta \right) \]

\[ \omega_{\text{LO}} = \frac{1}{R_1 C_1} \]
Cosine, Sine Signals in Receiver

They are used for down conversion
Pure I, Q signal generation

3\textsuperscript{rd}-order harmonics rejection

\[ I_{in} = \cos(\omega_{LO}t) + B \cos^3(\omega_{LO}t) \]

\[ Q_{in} = \sin(\omega_{LO}t) + B \sin^3(\omega_{LO}t) \]

With 3\textsuperscript{rd}-order harmonics.

Without 3\textsuperscript{rd}-order harmonics.

\[ I_{out} = A \cos(\omega_{LO}t + \theta) \]

\[ Q_{out} = A \sin(\omega_{LO}t + \theta) \]
Simulation of 3\textsuperscript{rd}-order harmonics rejection

\begin{align*}
I_{in}(t) &= \cos(\omega_{LO} t) + a \cos^3(\omega_{LO} t) \\
Q_{in}(t) &= \sin(\omega_{LO} t) + a \sin^3(\omega_{LO} t)
\end{align*}

\[ 3\omega_{LO} = \frac{1}{R_1 C_1} \]

\begin{align*}
I_{out}(t) &= A \cos(\omega_{LO} t + \theta) \\
Q_{out}(t) &= A \sin(\omega_{LO} t + \theta)
\end{align*}
Image Rejection Filter

\[ I_{in} = (A+B) \cos(\omega t) \]
\[ Q_{in} = (A-B) \sin(\omega t) \]
\[ I_{out} = A \cos(\omega t) \]
\[ Q_{out} = A \sin(\omega t) \]

\[ Ae^{j\omega t} + Be^{-j\omega t} \quad \rightarrow \quad Ae^{j\omega t} \]
Problem when $\omega_{LO} \neq 1/R_1C_1$

\[ \omega_{LO} = \frac{1}{R_1C_1} \]

\[ \omega_{LO} = \frac{2}{R_1C_1} \]

Amplitudes of $I, Q$ signals differ significantly.
The problem of large difference between $I_{out}$, $Q_{out}$ amplitudes can be alleviated

$$\omega_{LO} = \frac{2}{R_1 C_1}$$
The amplitude difference problem is further alleviated.

$$\omega_{LO} = \frac{2}{R_1 C_1}$$
Transfer Function Derivation
of RC Polyphase Filter
Transfer Function of RC Polyphase Filter

- Complex Signal Theory

- Complex input
- Complex output

- Complex Transfer Function

\[ V_{in}(j\omega) = I_{in} + j \cdot Q_{in} \]
\[ V_{out}(j\omega) = I_{out} + j \cdot Q_{out} \]
\[ G(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \]
Transfer Function of 1\textsuperscript{st}-order RC Polyphase Filter

Differential signal

\[ I_{in}(t) = I_{in+}(t) - I_{in-}(t) \]
\[ Q_{in}(t) = Q_{in+}(t) - Q_{in-}(t) \]
\[ I_{out}(t) = I_{out+}(t) - I_{out-}(t) \]
\[ Q_{out}(t) = Q_{out+}(t) - Q_{out-}(t) \]

Complex signal

\[ V_{in}(t) = I_{in}(t) + jQ_{in}(t) \]
\[ V_{out}(t) = I_{out}(t) + jQ_{out}(t) \]
Transfer Function of 1st-order RC Polyphase Filter

- **Transfer Function**

  \[ G_1(j\omega) = \frac{1 + \omega RC}{1 + j\omega RC} \]

- **Gain**

  \[ |G_1(j\omega)| = \frac{|1 + \omega RC|}{\sqrt{1 + (\omega RC)^2}} \]
Nyquist Chart of $G_1(j\omega)$

$$G_1(j\omega) = X(\omega) + jY(\omega)$$

Symmetric with respect to a line of $Y = -X$. 
Explanation of I, Q signal generation by $G_1(j\omega)$

$Q_{in}(t) \equiv 0, I_{in}(t) = \cos(\omega t)$,

$V_{in}(t) = \cos(\omega t) = \frac{1}{2}[e^{j\omega t} + e^{-j\omega t}]$

$|G_1(j\omega)|_{\omega=-\frac{1}{RC}} = 0, |G_1(j\omega)|_{\omega=\frac{1}{RC}} = \sqrt{2}, \angle G_1(j\omega) = -\frac{\pi}{4}$

$V_{out}(t) = \frac{1}{2}[|G_1(j\omega)|e^{j(\omega t+\angle G_1(j\omega))} + |G_1(-j\omega)|e^{j(-\omega t+\angle G_1(-j\omega))}]$

$= \frac{\sqrt{2}}{2}\cos(\omega t - \frac{\pi}{4}) + j\frac{\sqrt{2}}{2}\sin(\omega t - \frac{\pi}{4})$
Output Load ($C_L$) Effects

$$G_1(j\omega) = \frac{1 + \omega R_1 C_1}{1 + j\omega R_1 (C_1 + C_L)}$$
Input Impedance $Z_{in} =$

Complex Input Voltage

Complex Input Current

$Z_{in} = \frac{1+j\omega R_1 C_1}{2 j \omega C_1 [1+j(1+\omega R_1 C_1)]}$
Component Mismatch Effects

Mismatches $\delta X$ among R’s, C’s

Image signal $V_{out}$ is caused.

$$V_{out} = G_1 \cdot Vin + E_1 \cdot \delta X \cdot Vin$$

where

$$E_1 := \frac{(1 + j)R_1C_1\omega}{2(1 + jR_1C_1\omega)^2}$$

$G = \frac{V_{out}}{V_{in}}$
Transfer Function of 2\textsuperscript{nd}-order RC Polyphase Filter

Transfer Function

\[ G_2(j\omega) = \frac{(1 + \omega R_1 C_1)(1 + \omega R_2 C_2)}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2 R_1 C_2)} \]

Derivation is very complicated, so we used "Mathematica."

Gain \(|G_2(j\omega)|\) characteristics
Nyquist Chart of $G_2(j\omega)$

Symmetric with respect to a line of $X = 0$.

$G_2(j\omega) = X(\omega) + jY(\omega)$
Features of 2\textsuperscript{nd}-order RC Polyphase Filter

\[
|G_2(j\omega)| \neq |G_2(-j\omega)|, \quad |G_2(j\omega_1)| = |G_2(j\omega_2)|,
\]
\[
|G_2(-j\omega_1)| = |G_2(-j\omega_2)| = 0,
\]
\[
\lim_{\omega \to \pm \infty} |G_2(j\omega)| = 1, \quad |G_2(j0)| = 1,
\]
\[
\left[ \frac{\partial |G_2(j\omega)|}{\partial \omega} \right]_{\omega = \sqrt{\omega_1 \omega_2}} = 0, \quad \frac{|G_2(j\sqrt{\omega_1 \omega_2})|}{|G_2(-j\sqrt{\omega_1 \omega_2})|} = \left( \frac{\sqrt{\omega_1} + \sqrt{\omega_2}}{\sqrt{\omega_1} - \sqrt{\omega_2}} \right)^2
\]

For arbitrary $a$, \[
|G_1(ja\sqrt{\omega_1 \omega_2})| = |G_1(j\sqrt{\omega_1 \omega_2} / a)|.
\]
Flat passband design of 2\textsuperscript{nd}-order RC Polyphase Filter

Passband: $\omega_1 \sim \omega_2$
Stopband: $-\omega_1 \sim -\omega_2$

where $\omega_1 := \frac{1}{R_1 C_1}$,
$\omega_2 := \frac{1}{R_2 C_2}$

- To make gain in passband flat,
  $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1 \omega_2})|$.

- Image Rejection Ratio = $\left(\frac{\sqrt{\omega_2} + \sqrt{\omega_1}}{\sqrt{\omega_2} - \sqrt{\omega_1}}\right)^2$
Explanation why a 2\textsuperscript{nd}-order filter reduces I, Q amplitude difference.

Input signal

\[ Q_{in}(t) \equiv 0, \quad I_{in}(t) = \cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t}) \]

Output of a 1\textsuperscript{st}-order filter

\[ I_{out1}(t) + jQ_{out1}(t) = \frac{1}{2}[|G_1(j\omega)|e^{j\alpha + \theta_1} + |G_1(-j\omega)|e^{-j\alpha - \theta_1}] \]

\[ = |G_1(j\omega)|(1 + \frac{|G_1(-j\omega)|}{|G_1(j\omega)|})\cos(\omega t + \theta_1) + j|G_1(j\omega)|(1 - \frac{|G_1(-j\omega)|}{|G_1(j\omega)|})\sin(\omega t + \theta_1) \]
Explanation why a 2\textsuperscript{nd}-order filter reduces I, Q amplitude difference.

Output of a 2\textsuperscript{nd}-order filter

\[ I_{\text{out}2}(t) + jQ_{\text{out}2}(t) = \frac{1}{2} \left[ |G_2(j\omega)| e^{j\alpha + \theta_2} + |G_2(-j\omega)| e^{-j\alpha - \theta_2} \right] \]

\[ = |G_2(j\omega)| \left( 1 + \frac{|G_2(-j\omega)|}{|G_2(j\omega)|} \right) \cos(\omega t + \theta_2) + j |G_2(j\omega)| \left( 1 - \frac{|G_2(-j\omega)|}{|G_2(j\omega)|} \right) \sin(\omega t + \theta_1) \]

- According to the transfer functions,

\[ \frac{|G_1(-j\omega)|}{|G_1(j\omega)|} >> \frac{|G_2(-j\omega)|}{|G_2(j\omega)|} \]

at \( \omega \approx \omega_{\text{LO}} \)

then, the amplitude difference is reduced.
Transfer Function of 3\textsuperscript{rd}-order RC Polyphase Filter

\[ G_3(j\omega) = \frac{N_3(j\omega)}{D_{3R}(\omega) + jD_{3I}(\omega)} \]

where

\[ N_3(j\omega) = (1 + \omega R_1 C_1)(1 + \omega R_2 C_2)(1 + \omega R_3 C_3) \]

\[ D_{3R}(\omega) = \]

\[ 1 - \omega^2 \left[ R_1 C_1 R_2 C_2 + R_2 C_2 R_3 C_3 + R_1 C_1 R_3 C_3 + 2R_1 C_3 (R_2 C_2 + R_2 C_1 + R_3 C_2) \right] \]

\[ D_{3I}(\omega) = \]

\[ \omega \left[ R_1 C_1 + R_2 C_2 + R_3 C_3 + 2(R_1 C_2 + R_2 C_3 + R_1 C_3) \right] - \omega^3 R_1 C_1 R_2 C_2 R_3 C_3 \]
Gain, Phase of 3rd-order RC Polyphase Filter

Gain:
\[ |G_3(j\omega)| = \frac{|N_3(j\omega)|}{\sqrt{D_{3R}(j\omega)^2 + D_{3I}(j\omega)^2}} \]

Phase:
\[ \tan(\angle G_3(j\omega)) = -\frac{D_{3I}(j\omega)}{D_{3R}(j\omega)} \]

Gain characteristics
Nyquist Chart of $G_3(j\omega)$

In case

$R_1C_1 = R_2C_2 = R_3C_3$

Symmetric with respect to a line of $Y = X$.

$G_3(j\omega) = X(\omega) + jY(\omega)$
Summary

● Explicit transfer functions of 1st-, 2nd- and 3rd-order RC polyphase filters. Systematic design and analysis are possible.

● On-going projects
  - Derivation of higher-order filter ones.
  - Nonideality effects in higher-order filters.
  - Systematic design method using Nyquist chart.