A High-Precision AC Wheatstone Bridge Strain Gauge

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ABSTRACT
This paper describes high-precision dynamic strain measurement bridge circuits with on-line calibration of parasitic capacitance effects. The proposed calibration system is very reliable and robust against temperature change and aging because most of the calibration is done in digital domain.

Keywords: Strain Measurement, Strain Gauge, Bridge Circuit, Calibration.

1. Introduction

Recently much attention is being paid to sensor technology for automotive applications. In this paper we focus on high-precision strain measurement technology (Figs. 2 and 3, [1]-[8]) for such applications. Strain measurement methods can be classified into DC (Fig.4) and AC (Fig.5) methods. DC methods are simple, but suffer from low-frequency noise (such as 50Hz or 60Hz hum noise from power supply), drift, and thermal electromotive force (emf), and hence cannot achieve high precision. On the other hand, AC methods suffer from parasitic capacitance effects [1,2], even though they are not affected by low-frequency noise, drift, and thermal emf. In this paper, we describe how we have attempted to solve this problem of parasitic capacitance effects, which have been a problem for a long time in AC methods of strain measurement, using ADC and modern digital technology.

2. Principle of Strain Measurement: with Strain Gauge and Bridge Circuit

When a material is stretched (or compressed), the force used generates a corresponding stress \( \sigma \) inside the material. This stress in turn generates a proportional tensile strain (or compressive strain) which deforms the material by \( L + \Delta L \) (or \( L - \Delta L \)), where \( L \) is the original length of the material. When this occurs, the strain is the ratio of \( \Delta L \) to \( L \) (Fig.1).

Fig. 2. Strain gauge and Wheatstone bridge circuit (quarter bridge 2-wire system).

Fig.2 shows an example of a strain gauge and a Wheatstone bridge circuit (There are several types of their combinations according to applications). Suppose that \( V_{in} = 2[V] \) is applied, and further suppose
that the applied strain changes the gauge resistance from $R$ to $R + \Delta R$.

Then the Wheatstone bridge circuit output voltage $\Delta V$ is given by

$$\Delta V = \frac{\Delta R}{2R + \Delta R} = \frac{\Delta R}{2R + \frac{\Delta R}{R}} [V],$$

and we can obtain the strain value $\varepsilon$ as

$$\varepsilon = \frac{2}{k} \frac{\Delta V}{1 - \Delta V} \varepsilon \simeq \Delta V$$

where $\Delta V = \frac{k \varepsilon}{2 + k \varepsilon} [V]$.

3. Analysis of Parasitic Capacitance Influences

In applications where the strain gauge must be at some distance from the strain measuring instrument, so long wires must be used to connect them, the parasitic capacitance associated with the wires limits the accuracy of the measurement. In this section we analyze the parasitic capacitance effects. Fig. 6 shows a bridge circuit with parasitic capacitances, and its transfer function $H(j\omega)$ is given by:

$$H(j\omega) = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

where

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2}$$

The transfer function $H(j\omega)$ can be rewritten as

$$H(j\omega) = \frac{H_N(j\omega)}{H_D(j\omega)}$$

where

$$H_N = R_2^2 - R_1^2 + \omega^2 R_1^2 R_2^2 (C_1^2 - C_2^2)^2 + 2\omega R_1 R_2 (R_1 C_1 - R_2 C_2),$$

$$H_D = (R_2 + R_1)^2 + \omega^2 R_1^2 R_2^2 (C_1 + C_2)^2.$$
Fig. 7. SPICE simulation results of input and output waveforms in the bridge circuit with parasitic capacitances in Fig.6.

Fig. 8. Effect of parasitic capacitance effects on the real part of $H(j\omega)$.

Fig. 9. An analog method for parasitic capacitances $C_1$ compensation.

4. Conventional Strain Measurement Method

Conventional AC dynamic strain measurement systems measure the real part $H_R(j\omega)$ of the bridge output voltage by phase detection and calculate the strain, assuming that $C_1, C_2$ are small enough to be neglected, and also that $\epsilon^2$ and $\epsilon^3$ can be neglected. However, in recent applications where parasitic capacitances are not negligible and very high precision measurement is demanded, these assumptions are not valid any more. Sometimes an analog calibration method is used to compensate for parasitic capacitances as shown in Fig.9, but due to temperature change and aging effects this method cannot satisfy more demanding requirements.

5. Proposed Parasitic Capacitance Cancellation System

5.1 Configuration

Fig.10 shows our proposed strain measurement system, which has the following features:

(a) Oscillators of two different frequencies ($\omega_1, \omega_2$) are used. ($\omega_1/2\pi, \omega_2/2\pi$) are on the order of 10kHz.
(b) The bridge output is amplified by an AC amplifier which does not suffer from low frequency noise.
(c) The input and output signals of the bridge circuit are converted to digital data (with accuracy of 16 bits or greater) by delta-sigma AD modulators, and are stored in memory.
(d) A digital signal processor compensates for parasitic capacitance and calculates $\epsilon$ taking into account $\epsilon^2, \epsilon^3$ effects.

$$ H_R(j\omega) = R_2^2(1+k\epsilon\sigma)^2 - R_1^2(1+k\epsilon)^2 + \omega^2 R_1^2 R_2^2(1+k\sigma)^2(1+k\epsilon\sigma)^2(C_1^2 - C_2^2) + 2\omega^2 R_1 R_2(1+k\epsilon)(1-k\sigma)\{R_1 C_1(1+k\epsilon) - R_2 C_2(1-k\epsilon\sigma)\}, $$

$$ H_I(j\omega) = \{R_2^2(1-k\epsilon\sigma) + R_1(1+k\epsilon)^2\}^2 + \omega^2 R_1^2 R_2^2(1+k\epsilon)^2(1-k\epsilon\sigma)^2(C_1 + C_2)^2. $$

We see that both the real and imaginary parts of $H(j\omega)$ are affected by parasitic capacitances $C_1, C_2$ (Fig.8) and they are also functions of $\epsilon, \epsilon^2$ and $\epsilon^3$. 

ratio (a given value), then resistor values $R_1, R_2$ in the above equation for are replaced as follows

$$ R_1 \rightarrow R_1(1+k\epsilon), \quad R_2 \rightarrow R_2(1-L\epsilon), \quad L = k\sigma. $$

Then $H(j\omega)$ can be rewritten as

$$ H(j\omega) = H_R(j\omega) + jH_I(j\omega), $$

where
In other words, if the output presented in the digital domain, we have the value of
and taking the time average of the multiplication

\begin{align*}
\text{cos}(\omega_1 t) \times V_{\text{out}}(t) &= \frac{a_1}{2} + \frac{a_1}{2}\cos(2\omega_1 t) \\
&+ \frac{b_1}{2}\sin(2\omega_1 t),
\end{align*}

then \( H_R(j\omega_1) = \frac{a_1}{2} = V_{cr1}. \)

(2) In the same way, the input \( \cos(\omega_1 t) + \cos(\omega_2 t) \)

is applied to the bridge circuit, and its input and output are converted to digital signals. Digital filter operation is performed to them so that the component of \( \cos(\omega_1 t) \) is extracted from the input part and also \( \omega_1 \) component is obtained from the output. The input \( \cos(\omega_1 t) \) is phase-shifted by 90 degrees in the digital domain, to obtain \( \sin(\omega_1 t) \) precisely. (Accurate 90-degree phase-shift in digital domain is one of advantages of our proposed system.) Then by multiplying \( \sin(\omega_1 t) \) and the output \( \omega_1 \) component, and taking the time average of the multiplication result in the digital domain, we have the value of \( H_I(j\omega_1) = V_{ci}. \)

\[
\sin(\omega_1 t) \times V_{\text{out}}(t) = \frac{b_1}{2} - \frac{b_1}{2}\cos(2\omega_1 t) \\
+ \frac{a_1}{2}\sin(2\omega_1 t),
\]

then \( H_I(j\omega_1) = \frac{b_1}{2} = V_{ci}. \)

(3) Next we consider \( \omega_2 \) component. The input \( \cos(\omega_1 t) + \cos(\omega_2 t) \)

is applied to the bridge circuit, and its input and output are converted to digital signals. Digital filter operation is performed to them so that the component of \( \cos(\omega_2 t) \) is extracted from the input part and also \( \omega_2 \) component is obtained from the output. By multiplying the input and the output, and taking the time average of the multiplication result in the digital domain, we have the value of \( H_R(j\omega_2) = V_{cr2}. \)

(4) Then we have the following three equations

\begin{align*}
V_{cr1} &= \frac{R_2^2(1 - k\varnothing_1)^2 - R_1^2(1 + k\varnothing_1)^2 + \omega_2^2 R_1^2 R_2^2(1 - k\varnothing_1)^2(1 - k\varnothing_2)^2(C_1 - C_2)^2}{R_2(1 - k\varnothing_1) + R_1(1 + k\varnothing_1)^2 + \omega_2^2 R_1^2 R_2^2(1 + k\varnothing_1)^2(1 - k\varnothing_2)^2(C_1 + C_2)^2} \quad (1) \\
V_{cr2} &= \frac{R_2^2(1 - k\varnothing_2)^2 - R_1^2(1 + k\varnothing_2)^2 + \omega_2^2 R_1^2 R_2^2(1 - k\varnothing_1)^2(1 - k\varnothing_2)^2(C_1 - C_2)^2}{R_2(1 - k\varnothing_2) + R_1(1 + k\varnothing_2)^2 + \omega_2^2 R_1^2 R_2^2(1 + k\varnothing_2)^2(1 - k\varnothing_2)^2(C_1 + C_2)^2} \quad (2) \\
V_{ci} &= \frac{2\omega_1^2 R_1 R_2(1 + k\varnothing_2)(1 - k\varnothing_1)(1 + k\varnothing_2)(1 - k\varnothing_1) R_1 C_1(1 + k\varnothing_2)(1 - k\varnothing_1)}{R_2(1 - k\varnothing_2) + R_1(1 + k\varnothing_1)^2 + \omega_2^2 R_1^2 R_2^2(1 + k\varnothing_2)^2(1 - k\varnothing_2)^2(C_1 + C_2)^2} \quad (3)
\end{align*}

Since the number of unknown parameters \( (\varnothing, C_1, C_2) \) is three and there are three equations, this problem can be solved.

Fig. 10. Proposed dynamic strain measurement system.

5.2 Algorithm

The strain calculation algorithm is as follows:

(1) The input \( \cos(\omega_1 t) + \cos(\omega_2 t) \) is applied to the bridge circuit, and its input and output are converted to digital signals. Digital filter operation is performed to them so that the component of \( \cos(\omega_1 t) \) is extracted from the input part and also \( \omega_1 \) component is obtained from the output. By multiplying the input \( \cos(\omega_1 t) \) and output \( \omega_1 \) component signal, and taking the time average of the multiplication result in the digital domain, we have the value of \( H_R(j\omega_1) = V_{cr1}. \)

In other words, if the output \( \omega_1 \) component signal \( V_{out1}(t) = a_1\cos(\omega_1 t) + b_1\sin(\omega_1 t), \)

\[
\cos(\omega_1 t) \times V_{\text{out}}(t) = \frac{a_1}{2} + \frac{a_1}{2}\cos(2\omega_1 t) \\
+ \frac{b_1}{2}\sin(2\omega_1 t),
\]

then \( H_R(j\omega_1) = \frac{a_1}{2} = V_{cr1}. \)
Digital domain calculation enables taking care of $\varepsilon^2$, $\varepsilon^3$ terms. Our numerical calculations with typical parameter values show that ignoring $\varepsilon^3$ terms causes about 0.001% error, and ignoring both $\varepsilon^2$ and $\varepsilon^3$ terms causes about 0.14% error. Hence, for greater than 16-bit accuracy, both terms must be taken into account. If we consider $\varepsilon$, $\varepsilon^2$ and $\varepsilon^3$ terms, the above equations are a third-order polynomial in $\varepsilon$, and hence we have three possible solutions for $\varepsilon$. However, we have developed an algorithm which chooses the correct value from the three values.

6. Advantages

Parasitic capacitance values are unknown and can change according to temperature and aging. Analog signal processing circuits may generate additional noises inside them. However the above-mentioned system and algorithm are digital, and they can reliably compensate for parasitic capacitance on-line. Delta-sigma ADCs with greater than 16-bit accuracy and a few tens kilo Hertz bandwidth are now commercially available, and which, together with recent rapid progress of DSP, enable realization of our proposed system at low cost.

7. Conclusions

We have proposed a high-precision strain measurement system based on ADC and digital technology. The effectiveness of this system was investigated by simulation and by numerical calculation. Next we plan to implement the proposed system.

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References


Appendix

This appendix describes a conventional analog method to compensate parasitic capacitances $C_1$ (Fig.9). $C_1$ effects are compensated when the ratio of $r_1$ to $r_2$ is adjusted so that the current $I_e$ which flows through $C_1$ is equal to $I_1$, and then $I_2$ and the current which flows through $R_1$ are equal. (In other words, parasitic capacitance $C_1$ is compensated by $C_e$, $r_1$, $r_2$ and the operational amplifier). Currents $I_1$, $I_2$, $I_3$ and voltages $V_D$, $V_M$ are given by

$$I_1 = I_2 + I_3, \quad I_1 = \frac{V_m - V_B}{Z_1}, \quad I_2 = \frac{V_B}{R_2}, \quad I_3 = \frac{V_B - V_M}{Z_e},$$

$$V_M = \frac{r_1}{r_1 + r_2} V_m, \quad V_B = \frac{R_2 + j\omega R_1 R_2 (C_1 + \alpha C_e)}{R_1 + R_2 + j \omega R_1 R_2 (C_1 + C_e)} V_m.$$  

$$(\therefore z_1 = \frac{R_1}{1 + j \omega R_1 C_1}, \quad z_e = \frac{1}{j \omega C_e})$$  

Hence, the current $I_e$ which flows $C_1$ is given by

$$I_e = \frac{R_1 + j \omega R_1 R_2 C_e (1 - \alpha)}{R_1 + R_2 + j \omega R_1 R_2 (C_1 + C_e)} j \omega C_1 V_m.$$  

Also $I_3$ is can be rewritten as

$$I_3 = \frac{-\alpha R_1 R_2 (1 - \alpha)}{R_1 + R_2 + j \omega R_1 R_2 (C_1 + C_e)} j \omega C_2 V_m.$$  

Here, $\alpha = r_2/(r_1 + r_2)$. Then we can derive that the condition for $I_e = I_3$ is

$$r_2 = \frac{C_e R_e - C_1 R_1}{R_1 (C_e + C_1)}.$$  

With this condition, the parasitic capacitance $C_1$ effects are compensated. However, the value of $C_1$ may be unknown and also it may change according to temperature and aging, and furthermore automatic adjustment of the ratio $r_2/r_1$ is difficult; thus its complete cancellation with this analog method is very difficult.